

Reduction of DoA Estimation Errors Caused by Antenna Array Imperfections

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Abstract - Antenna arrays are expected to be widely used for future mobile radio applications. Therefore implementation issues are a matter of concern. Since real antenna arrays suffer from different impairments, sophisticated calibration procedures are required in order to enhance the performance. For that aim a promising calibration approach is described. Its performance has been verified by simulations and measurements. Especially, the application for the direction of arrival estimation in wideband radio vector channel sounders is addressed.

I. INTRODUCTION

Wideband radio vector channel sounders (WRVCS) using antenna arrays are usually applied for the acquisition of the time-varying, directional complex channel impulse response. To improve the direction of arrival (DoA) estimation accuracy and resolution capability, superresolution techniques are applied. Field tests, however, have shown that these methods are very sensitive against measurement errors that result from different antenna array imperfections. Consequently, there is a demand for high precision calibration methods.

After discussing issues of antenna array imperfections, an efficient calibration approach is outlined that is based on [1]. The results of simulations as well as measurements, carried out with the *RUSK* WRVCS [2], are presented. For DoA estimation we consider the unitary ESPRIT [3] because of its high computational efficiency.

II. ANTENNA ARRAY IMPERFECTIONS

Uniform linear arrays (ULA) are very popular for DoA estimation. Since the antenna elements are spaced by only about half a wavelength, strong mutual electro-magnetic coupling occurs. This leads to severe non-uniform distortions of the antenna element beam patterns. The ESPRIT, however, relies on identical beam patterns. Any non-uniform beam pattern ripple is for example misinterpreted as an outcome of some phantom wavefront. That reduces the dynamic range, the accuracy, the resolvability of closely spaced sources and may even cause wrong estimates.

Besides the mutual coupling, there is a variety of impairments such as geometrical and electrical tolerances

that cause the individual antennas to show up non-uniform amplitude and phase characteristics of the beam patterns. Examples are manufacturing inaccuracies, edge effects because of the finite size of the array and the ground plane, parasitic reflection and diffraction of components fixed w.r.t. the array, imperfect matching of the antenna outputs, etc. Another disturbing effect which is not really caused by the antenna array itself results from differences in the electrical feeder cable lengths. Fig. 1 shows the resulting beam patterns from a wide-band measurement. If measured separately from the array, a single element would show a much smoother characteristic than depicted in the figure. The non-uniform, ripple-like behavior mainly results from the arrangement in the array environment. The aim of this paper is to demonstrate the performance of the proposed calibration method by using simulated as well as measured array responses. It will be investigated if the broadband performance of the calibration result can be enhanced by using frequency-dependent sets of calibration parameters.

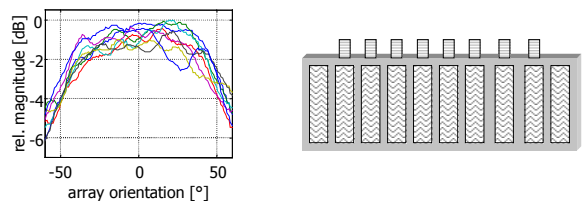


Fig. 1 Measured beam patterns (left) and ULA structure (right)

The ULA used for the measurements contains 8 active elements and 2 passive, terminated edge elements as indicated in Fig. 1. To reduce the influence of environmental reflections, the measurement has been performed in an anechoic chamber. In order to selectively analyze the influences of mutual coupling and finite size as well as to separate them from other parasitic effects, that cannot be completely avoided, an electro-magnetic field simulation has been performed. But in contrast, the simulated structure contains only 8 elements. Therefore, in order to remove the disturbing influence of the edge elements, only 6 antennas are used in order to calculate the error measures in the following. Fig. 2 and Fig. 3 respectively show the simulated and measured response for the carrier frequency of 5.2 GHz. Here only the beam pattern ripple is indicated, since as discussed above, the mean beam pattern shape is of less importance to the final ESPRIT result.

It should be noticed that all of the effects discussed here can be considered as being time-invariant. Therefore, there is no need for online calibration. This is opposed to amplitude and phase drift calibration of down-converter channels in multi-channel array receivers which is not considered here. A typical application is the channel sounder described in [2] since it relies on fast antenna switching and uses only one single down-converter channel.

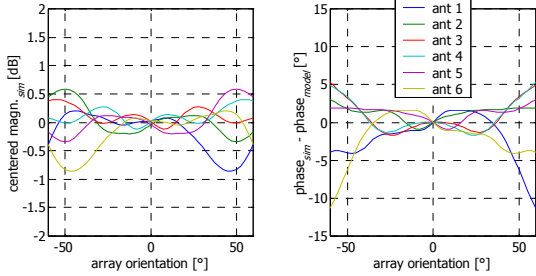


Fig. 2 Simulated 6 element ULA beam pattern ripple (mag. and phase)

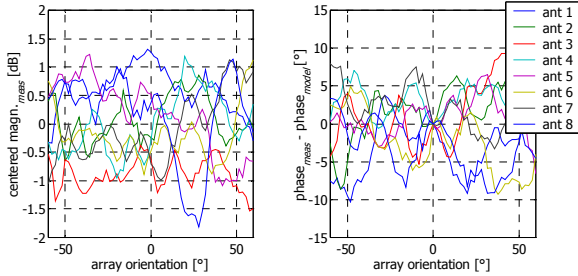


Fig. 3 Measured 8 element ULA beam pattern ripple (mag. and phase)

III. CALIBRATION

The basic idea of the applied calibration algorithm has been described in [1] and was adopted for channel sounder application in [4].

A. Procedure

The calibration measurement procedure is based on a set of N reference measurements in a well-defined anechoic environment. A reference antenna is placed on an equidistant grid of known azimuth angles θ_v with $v=1(1)N$, within the array azimuth sector of interest. Zero degree azimuth angle stands for the broadside direction. Actually, the reference antenna is kept fixed and the array is being rotated. A minimum distance between the Tx antenna and the Rx antenna array of approximately 100λ (λ : wavelength) is chosen to get nearly plane wavefronts at the antenna array [4].

B. Algorithm

If a single planar wavefront with the complex-valued pathweight γ from the direction θ_v impinges on an ULA (characterized by M elements spaced by the distance d) the array response vector results in $\mathbf{x}=\mathbf{a}\gamma$, with the array steering vector $\mathbf{a}(\theta_v)=\left[1 e^{-j2\pi\frac{d}{\lambda}\sin\theta_v} \dots e^{-j2\pi(M-1)\frac{d}{\lambda}\sin\theta_v}\right]^T$.

In a more realistic way the measured array output is modeled by:

$$\tilde{\mathbf{x}} = \mathbf{K}\mathbf{a}\gamma + \mathbf{n}. \quad (1)$$

The array imperfections are described by an $M \times M$ error matrix \mathbf{K} . This error matrix in its main diagonal directly describes amplitude and phase misadjustments of the antennas. The secondary diagonals cause mutual output correlation introduced, e.g., by electro-magnetic coupling between the different antennas. The aim of the calibration algorithm is to find a calibration matrix $\mathbf{K}_{cal}=\mathbf{K}^{-1}$ that effectively removes the systematic error if applied to the array output. The proposed algorithm to estimate \mathbf{K}_{cal} is based on the idea that for an error-free array a set of orthogonal null steering vectors $\mathbf{c}_\mu(\theta_v)=\mathbf{a}_\mu(\theta_v)e^{-j2\pi\mu/M}$ with $\mu=1(1)M-1$ exists. They span the equivalent nullspace of the reference source response vector $\mathbf{a}(\theta_v)$. In the measured case that nulling property $0=\mathbf{c}_\mu^H \mathbf{x} \mathbf{x}^H \mathbf{c}_\mu$ will no longer hold and with the calibration matrix \mathbf{K}_{cal} applied to $\tilde{\mathbf{x}}$, the remaining MSE:

$$\varepsilon^2 = \sum_{\mu=1}^{M-1} \mathbf{c}_\mu^H \mathbf{K}_{cal} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^H \mathbf{K}_{cal}^H \mathbf{c}_\mu \quad (2)$$

can be minimized. Since the array error model (1) does not completely match all physical sources of error and since measurement noise \mathbf{n} cannot be avoided, only an estimate $\hat{\mathbf{K}}_{cal}$ can be obtained. After applying a matrix-vector-exchange to $\hat{\mathbf{K}}_{cal} \Rightarrow \hat{\mathbf{k}}_{cal}$ and the reverse one to $\mathbf{c}_\mu \Rightarrow \mathbf{C}_\mu$, (2) can be rewritten, yielding:

$$\varepsilon^2 = \sum_{\mu=1}^{M-1} \hat{\mathbf{k}}_{cal}^H \tilde{\mathbf{R}}_\mu \hat{\mathbf{k}}_{cal} \quad \text{with} \quad \tilde{\mathbf{R}}_\mu = \mathbf{C}_\mu^H \tilde{\mathbf{x}} \tilde{\mathbf{x}}^H \mathbf{C}_\mu. \quad (3)$$

Averaging over a scenario of N different directions should be applied in order to enhance the rank of the covariance

matrix $\tilde{\mathbf{R}} = \sum_{v=1}^N \sum_{\mu=1}^{M-1} \hat{\mathbf{k}}_{cal} \tilde{\mathbf{R}}_{v\mu} \hat{\mathbf{k}}_{cal}^H$ and to further increase the

stability of the estimate $\hat{\mathbf{k}}_{cal}$ that minimizes $\tilde{\mathbf{R}}$. Finally the reverse matrix-vector-exchange to $\hat{\mathbf{k}}_{cal} \Rightarrow \hat{\mathbf{K}}_{cal}$ is required.

IV. CALIBRATION RESULTS

The purpose of this section is to demonstrate the performance of the calibration procedure under various circumstances. Two frequency-dependent sets of calibration matrices were determined for the simulated as well as the measured array responses. In order to show the frequency dependence of the calibration two cases have been compared. For the first one, only the calibration matrix $\hat{\mathbf{K}}_{cal}$ at the carrier frequency of 5.2 GHz has been applied for calibration resulting in frequency-independent coefficients of the calibration matrix $\hat{\mathbf{K}}_{cal}$. For the second one, the whole frequency-dependent set has been applied. For both cases, the result of the calibration is demonstrated vs. frequency and vs. DoA (array orientation). In case of the frequency dependent demonstration, the MSE is averaged over the angle of arrival, whereas in case of the array orientation dependent demonstration, the MSE is averaged over the frequency range. Moreover, the number of the used diagonals of the calibration matrix has been varied. With 1 diagonal used, this corresponds to simple

weighting of the antenna outputs. If the fully occupied calibration matrix with its 10 secondary diagonals (in case of 6 antennas) is used, that corresponds to combining of all antenna outputs. In general, it can be expected that the contribution of the outer secondary diagonals is gradually becoming smaller since the electro-magnetic coupling will be smaller for more distant antennas. To indicate the quality of the calibration two error measures are calculated. The first is the relative non-uniformity of the resulting beam pattern which is defined by the relative variance of the beams at a given DoA resp. source angle. The second measure is the r.m.s. error of the DoA's. They have been estimated by the unitary ESPRIT.

A. Simulation

The generation of the simulation data for a 6 element ULA with 2 additional passive side elements is based on a program for electro-magnetic modeling of composite and dielectric structures delivering frequency-dependent complex-valued beam patterns. The elements are modeled as dipoles fed by microstrip lines, separated by a thin loss-free substrate with a relative permittivity of 3.38. The microstrip lines are mounted on a substrate with the same relative permittivity. Below the substrate a finite sized metal ground plane is arranged. Between the elements absorber material is inserted with a relative permittivity of 1.3-j. During the simulation of the response of any single element all the other elements are terminated with 50Ω loads. The simulation bandwidth thereby was 120 MHz, the center frequency has been chosen as 5.2 GHz resulting in a relative bandwidth of 2.3%.

Fig. 4 below demonstrates the improvements for the beam patterns relative to Fig. 2 based on the estimated calibration matrix (full occupied, i.e. 11 diagonals). The applied calibration approach clearly shows the capability to reduce non-uniformities of the element beam patterns. Fig. 5 indicates the maximum magnitudes of the calibration matrix for a different number of diagonals used.

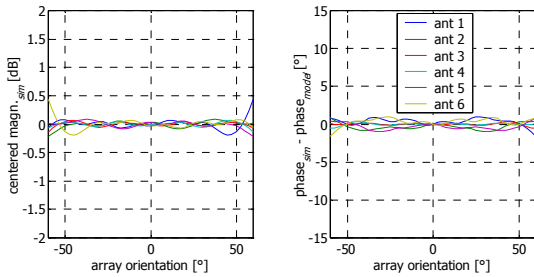


Fig. 4 Non-uniformity of the calibrated array beam patterns

The results of both simulations are compared in Fig. 6 and Fig. 7. The right figures show the result for the frequency-independent calibration matrix and the left figures accordingly those of the frequency-dependent one. Fig. 6 depicts the remaining relative non-uniformity of the beam patterns averaged over the array orientation (upper plot) and over the frequency (lower plot). The respective angle estimation errors using the unitary ESPRIT are plotted in Fig. 7.

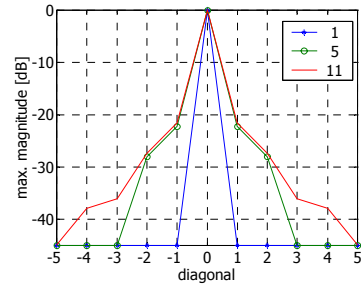


Fig. 5 Maximum magnitude within the diagonals of the calibration matrix (1,5,11 diagonals)

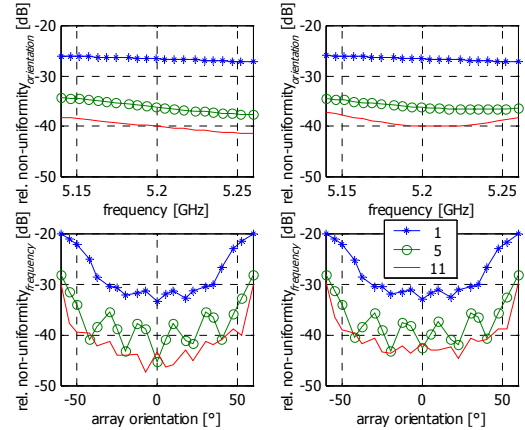


Fig. 6 Relative non-uniformity after calibration (1,5,11 diagonals)

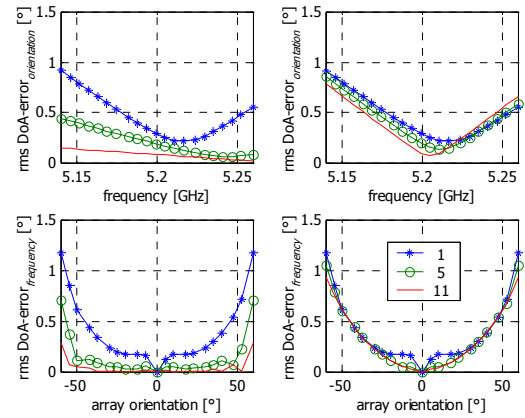


Fig. 7 r.m.s. DoA estimation errors (1,5,11 diagonals)

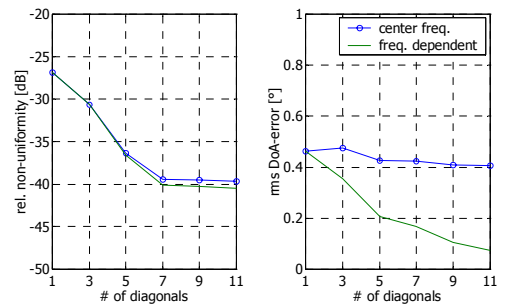


Fig. 8 Relative non-uniformity (left) and the r.m.s. DoA error (right)

The dependence of the relative non-uniformity and the respective angle estimation error integrated over frequency and incident angle vs. the used secondary diagonals are shown in Fig. 8.

B. Measurement

The measurement data are based on a measurement campaign in an anechoic chamber with the *RUSK* WRVCS utilizing an 8 element ULA with 2 additional passive side elements operating at 5.2 GHz with a bandwidth of 120 MHz. The following results were prepared similar like explained in the subsection before.

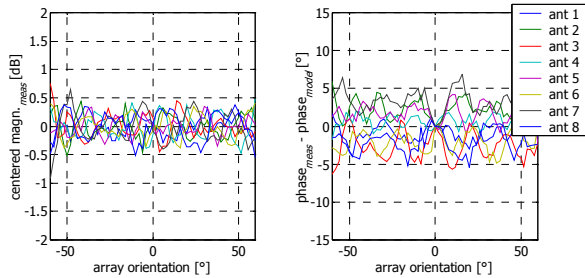


Fig. 9 Non-uniformity of the calibrated array beam patterns

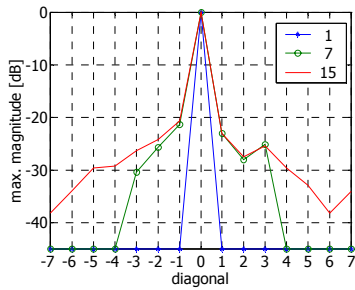


Fig. 10 Maximum magnitude within the diagonals of the calibration matrix (1,7,15 diagonals)

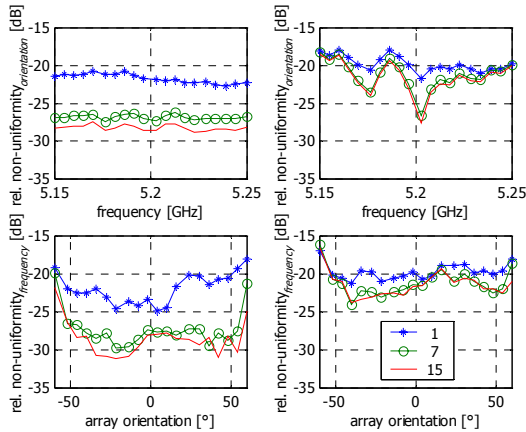


Fig. 11 Relative non-uniformity after calibration (1,7,15 diagonals)

V. CONCLUSIONS

In this paper a recently proposed calibration approach for antenna arrays has been analyzed for ESPRIT-type DoA estimation applications. Both simulated and measured array characteristics have been used to show that this approach considerably improves the array performance. That can be seen from a reduced mean squared deviation between the individual element beam patterns as well as from an increased accuracy of DoA's estimated by a unitary ESPRIT. The calibration approach consists of a single source reference measurement in an anechoic environment and of a least squares estimation of a

calibration matrix $\hat{\mathbf{K}}_{cal}$. The result is a calibration matrix that combines all array outputs. It has been shown that the best performance is achieved if all diagonals of the calibration matrix are used. But even more important is that a considerable performance improvement can be achieved if frequency dependent coefficients are applied. Therefore, different calibration data sets have to be used in the uplink and in the downlink in case of frequency duplex systems. Also for broadband channel sounder applications, where the paths are estimated in the time-delay domain, frequency dependent calibration coefficients should be used.

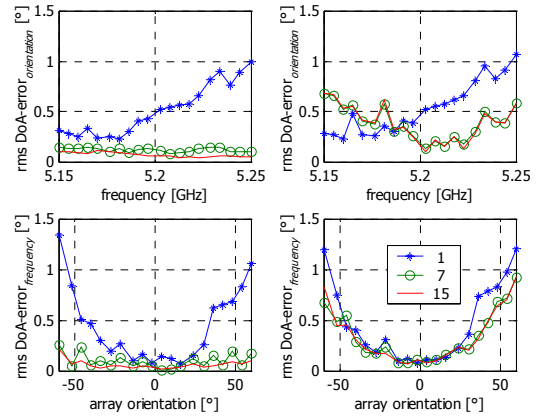


Fig. 12 r.m.s. DoA estimation errors (1,7,15 diagonals)

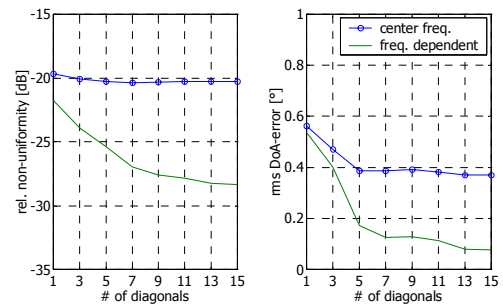


Fig. 13 Relative non-uniformity (left) and the r.m.s. DoA error (right)

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VI. REFERENCES

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