RECENT ADVANCES IN MODELING AND ESTIMATION OF LOCAL AND DISTRIBUTED SCATTERING OF MOBILE RADIO CHANNELS

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Abstract

In this article we introduce a stochastic parametric model for the scattered waves, which are frequently observed in channel sounding measurements. We also propose an estimator for the model parameters. The proposed model employs a mixture of angular von Mises distributions which is appropriate for directional data observed in channel measurement campaigns. The power-delay profile is modeled using an exponential distribution which is also typically observed in measurement campaigns. The parameters of the exponential and the von Mises distributions as well as the measurement noise power are estimated from data. This approach leads to a low complexity algorithm and faster convergence in comparison to pure deterministic models, like those based on the SAGE algorithm.

I. INTRODUCTION

In radio propagation it is usual to classify the signals that reach the receiver as been originated by specular reflections or scattering. The specular components usually carry most of the power, and are often modeled by a relatively large number of deterministic signals with unknown parameters. Diffuse scattering is frequently regarded as noise and neglected in the models. However, even though each scattered wave arrives with low power, the total power of scattering components can be significant, and even dominant in some scenarios [1].

Deterministic techniques for propagation parameter estimation commonly attempt to model the scattered waves with a large number of discrete waves. This approach leads to maximization of highly non-linear multidimensional likelihood functions with many local optima. This requires solving multidimensional optimization problems and causes convergence problems. The computational complexity and variance of estimates are increased, since parameters of a large number of waves need to be estimated.

In this article we present a stochastic parametric model for the scattered waves, and we also introduce an estimator for the model parameters. The proposed model employs angular von Mises distribution, which is appropriate for directional data observed in channel measurement campaigns. The angular distribution is modeled as a mixture of von Mises distributions, which correspond to scatterer clusters. Only few parameters are estimated from data, namely the parameters of the individual distributions, namely the mean angle and scatter, as well as the mixture proportions. The mixture model allows for representation of scenarios where multiple scatterer clusters are present, and also for representation of each cluster as a finite mixture of von Mises distributions [2].

The power-delay profile is modeled using an exponential distribution, which is typically observed in measurement campaigns. The parameters of the exponential distribution and the measurement noise power are estimated.

In order to estimate the scattered waves properly, it is necessary to remove the specular components from the measured signal. On the other hand, it is necessary to have an estimate of the covariance matrix of the scattered waves and noise for the estimation of the specular components. These requirements imply that both specular and scattered waves are to be estimated jointly, and hence we a stochastic maximum likelihood method with deterministic methods [3], like in the RIMAX algorithm. Due to the stochastic modeling of the diffuse scattering, a smaller number of specular components has to be estimated, resulting in smaller variance of the estimates and improved convergence. Also, this approach leads to lower complexity and faster convergence in comparison to pure deterministic models, like those based on the Space Alternating Generalized Expectation-maximization (SAGE) algorithm [4]. These benefits are due to a smoother likelihood function and reduced dimensionality of the problem, leading to lower variance of estimates, reduced computational complexity, and improved convergence.

II. SIGNAL MODEL

The base-band representation of the double-directional channel model [5] is used to describe the contribution of specular-alike (concentrated) propagation paths and a multivariate circular complex normal distributed process to describe the distributed diffuse scattering of the radio channel.

A. Model for Specular Propagation Paths

This data model is based on the assumption that every specular propagation path can be described as a R_p-dimensional (5-D) shift operator on the transmit signal. It shifts the Tx-signal in the 4 independent angular domains, i.e. transmit azimuth φ_T and elevation ϑ_T, receive azimuth φ_R and elevation ϑ_R, as well as in the time-delay domain τ. Furthermore, it is assumed that the 5 related aperture domains (frequency and antenna array apertures) are finite. The periodic excitation signal is band-limited, and the aperture of the antenna arrays is determined by their size, which is finite too. Finally, the parameters are or may be treated as bounded. All angles are bounded to $(-\pi, \pi)$, and the time-delay is bounded to $(0, \tau_{max})$ where τ_{max} is a function of transmit power, free space loss, and receiver noise. Under these conditions and considering that a shift in one domain can also be expressed by the multiplication with a complex exponential in the related aperture domain, the family of exponential functions is sufficient to construct a data model for a specular propagation path. For notational convenience we replace the shift-parameters of propagation path (component) p from the physical model using normalized shift parameters $\mu_p^{(r)}$, which are related to their physical counterparts $\varphi_{T,p}$, $\vartheta_{T,p}$, $\varphi_{R,p}$, $\vartheta_{R,p}$, and τ_p by a unique projection [5].

B. Model for Diffuse Scattering Components

In [5] it is shown, that the observed radio channel consists not only of specular components but also of diffuse scattering components (DSC). Hence, we approximate the radio channel observation $\mathbf{y} \in \mathbb{C}^{M\times 1}$ by the superposition of a finite number P of specular propagation paths and the realization of a stochastic process describing DSC and measurement noise. We assume that the complex vector $\mathbf{n}_d$, describing the distribution of the observed diffuse scattering components, is drawn from a multivariate circular complex white Gaussian process $\mathcal{N}_c\left(\mathbf{0}, \mathbf{R}_d\right) \in \mathbb{C}^{M\times 1}$. Furthermore, we model the measurement noise by a zero mean circular complex Gaussian process $\mathbf{n}_m \sim \mathcal{N}_c\left(\mathbf{0}, \sigma^2\mathbf{I}\right) \in \mathbb{C}^{M\times 1}$, where σ^2 denotes the noise variance. The complete model for a channel observation is given by

$$\mathbf{y} = \mathbf{s}\left(\boldsymbol{\theta}\right) + \mathbf{n}_d + \mathbf{n}_m. \tag{1}$$

The vector $\mathbf{n}_d$ can be decomposed as $\mathbf{n}_d = \mathbf{w} \otimes \mathbf{h}$, where $\mathbf{w} \in \mathbb{C}^{M_f\times 1}$ is a complex vector describing the delay- or frequency dependency of $\mathbf{n}_d$, M_f is the number of frequency samples, $\mathbf{h} \in \mathbb{C}^{M_r M_t\times 1}$ is a complex vector representing the spatial dependency of $\mathbf{n}_d$, M_r is the number of receive antennas, M_t is the number of transmit antennas, and $\otimes$ denotes the tensor (Kronecker) product. Both $\mathbf{w}$ and $\mathbf{h}$ are assumed to be independent and drawn from a multivariate circular complex Gaussian process $\mathcal{N}_c\left(\mathbf{0}, \mathbf{R}_w\right) \in \mathbb{C}^{M_f\times 1}$ and $\mathcal{N}_c\left(\mathbf{0}, \mathbf{R}_h\right) \in \mathbb{C}^{M_r M_t\times 1}$, respectively.

1) Delay and Frequency Domain Characterization: For the delay domain we use the model in [5], which is based on the observation that the power delay profile has an exponential decay over time and a base delay, which is related to the distance between the transmitter and receiver. Let us define the sampled version of the correlation function $\mathbf{v}(\boldsymbol{\theta}_w)$, $\boldsymbol{\theta}_w = \{\alpha_1, \beta_d, \tau_d\}$, in frequency-domain as

$$\mathbf{v}(\boldsymbol{\theta}_w) = \frac{\alpha_1}{M_f}\left[\frac{1}{\beta_d} \quad \cdots \quad \frac{e^{-j2\pi(M_f-1)\tau_d}}{\beta_d + j2\pi\frac{M_f-1}{M_f}}\right], \tag{2}$$

where τ_d is the normalized base delay, β_d is the normalized coherence bandwidth, and α_1 denotes the maximum power.

The covariance matrix of the diffuse scattering in frequency domain is modeled as a Toeplitz matrix

$$\mathbf{R}_w = \mathrm{toep}\big(\mathbf{v}(\boldsymbol{\theta}_w), \mathbf{v}(\boldsymbol{\theta}_w)^H\big). \tag{3}$$

2) Angular Domain Characterization: Using the extended Saleh-Valenzuela (SVA) channel model in [6] we can write $E[\mathbf{h}\mathbf{h}^H]$ as a function of the angular parameters. A similar model is obtained in [2], [7] following a different approach that is related to the geometry of the distribution of scatterers. For simplicity, we will limit the discussion to uni-directional estimation, but the results can be naturally extended to the double directional case. We also assume (for simplicity) that an uniform linear array (ULA) is used at the receiver. Then the correlation at the receiver side is given by

$$\mathbf{R}_{h,m_1m_2}(\boldsymbol{\theta}_h) = \int_{-\pi}^{\pi} \exp(b_{m_1m_2}\cos(\phi)) f(\phi, \boldsymbol{\theta}_h) d\phi, \tag{4}$$

where $f(\phi, \boldsymbol{\theta}_{h,l})$ is any angular PDF of ϕ and characterized by parameters $\boldsymbol{\theta}_h$, $b_{m_1m_2} = j2\pi d_{m_1m_2}$, and $d_{m_1m_2}$ is the distance between elements m_1 and m_2 in the receive array.

In channel measurements it is often found that signals are arriving from a number of different clusters, especially in bad urban scenarios. This is equivalent to a situation where the angular PDF $f(\phi)$ in equation (4) is a mixture of distributions,

$$f(\phi) = \sum_{p=1}^{P} \epsilon_p f_p(\phi), \tag{5}$$

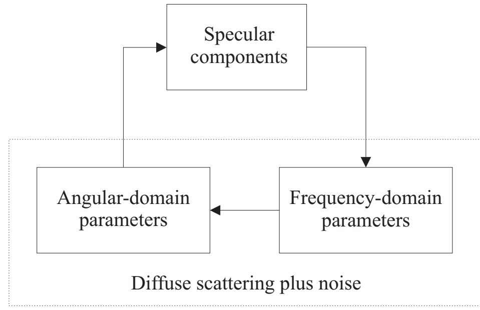


Fig. 1. Two-step procedure for joint optimization of specular components and diffuse scattering parameters.

where P is the number of clusters, $\sum_{p=1}^{P} \epsilon_p = 1$, ϵ_p are unknown mixture proportions, and $f_p(\phi)$ is any valid angular PDF. An angular PDF must satisfy $f(\theta) = f(\theta + 2\pi k)\ \forall\ k \in \mathbb{Z}$. Hence, a Gaussian PDF, which has infinite support, is not suitable. The von Mises distribution [8] defined in angular domain is more appropriate. It is defined as follows:

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)} \exp(\kappa \cos(\phi - \mu)), \tag{6}$$

where μ is the symmetry center or "mean angle", and κ is related to the variance of the von Mises distribution, i.e. how scattered about the symmetry center. κ can be chosen between 0 (isotropic scattering) and ∞ (extremely concentrated), and $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero.

With this definition of the angular PDF, the angular domain parameters are defined as $\boldsymbol{\theta}_h = \{\mu_p, \kappa_p, \mu_p\}$, $p = 1, \ldots, P$.

Using (6), the cross correlation in (4) may be written analytically as [2], [7]

$$\mathbf{R}_{h,m_1m_2}(\boldsymbol{\theta}_h) = \sum_{p=1}^{P} \epsilon_p \frac{I_0(\{\kappa_p^2 + b_{m_1m_2}^2 + 2\kappa_p b_{m_1m_2} \cos(\mu_l)\}^{\frac{1}{2}})}{I_0(\kappa_p)}. \tag{7}$$

III. Parameter Estimation

Let us denote by $\mathbf{y}_m$ the m-th observation of $\mathbf{y}$, $m = 1, \ldots, M_s$. Assuming $\mathbf{y}$ is circular complex Gaussian and that the realizations $\mathbf{y}_m$ are i.i.d., we can write the likelihood function as

$$L(\mathbf{y}_1, \ldots, \mathbf{y}_{M_s}) \propto -\log|\mathbf{R}_y| - \frac{1}{M_s} \sum_{m=1}^{M_s} \mathbf{y}_m^H \mathbf{R}_y^{-1} \mathbf{y}_m, \tag{8}$$

where M_s is the number of realizations or snapshots, and $\mathbf{C}_y$ is the covariance matrix of $\mathbf{y}$, given by

$$\mathbf{R}_y = E[(\mathbf{y} - \mathbf{s}(\boldsymbol{\theta}))(\mathbf{y} - \mathbf{s}(\boldsymbol{\theta}))^H] = \mathbf{R}_w \otimes \mathbf{R}_h + \sigma^2 \mathbf{I}. \tag{9}$$

Direct optimization for all parameters is not feasible due to the high number of parameters and due to the dimensions of the matrices involved. Instead we will use an iterative procedure:

(1) Optimize for the parameters of the specular components using the previously estimated covariance matrix.
(2) Remove the specular components from data and optimize for the covariance matrix of the diffuse scattering components plus noise variance.

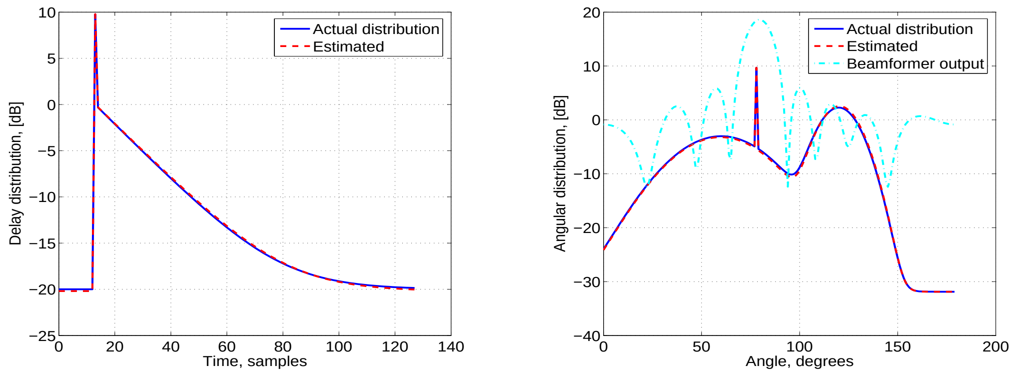
Step 2 can also be decomposed in two steps:

(2.a) Optimize for the frequency-domain parameters and noise variance.
(2.b) Optimize for the angular-domain parameters, with $\mathbf{R}_w$ as calculated in the previous step.

This iterative procedure is illustrated in Figure 1, where a block diagram of the iterative procedure is given.

The first benefit of the proposed method is the separate optimization of specular and diffuse scattering components, reducing the number of variables for each local optimization. A similar approach has been investigated in [5], where the DSC was assumed to be spatially white.

The further decomposition of step 2 into steps 2.a and 2.b is important due to the high dimensionality of the matrices involved. This decomposition has been proposed first in [3], where no further iterations of steps 2.a and 2.b were considered. Typical values for M_f and M_rM_t are in the range $M_f = [100, 2000]$, and $M_rM_t = [4, 64]$, but higher values are not uncommon. This leads to $\mathbf{R}_y$ ranging in dimension from 400×400 up to 128000×128000, or even higher. The decomposition is obtained by pre-whitening the observation vector such that the part of the covariance matrix corresponding to one of the domains becomes the identity matrix, keeping the other part unchanged. As a consequence, the noise covariance matrix is also changed, but no noise correlation is introduced. This transformation decouples both domains, making it possible to optimize for them separately. In step 2.a the Toeplitz structure of $\mathbf{R}_w$ is exploited to reduce computational complexity, and in step 2.b the reduction of complexity arises from the reduction in the size of the matrices involved from $M_fM_rM_t \times M_fM_rM_t$ to $M_rM_t \times M_rM_t$. Also, if $\mathbf{R}_h$ has a Toeplitz structure it can be explored to simplify further the computations.



(a) (b)

Fig. 2. Comparison of (a) estimated power delay profile and actual power delay profile, and (b) estimated power angular profile and actual power angular profile. The curves overlap almost perfectly. Also shown in (b) is the output of the Bartlett beamformer.

IV. Simulation Examples

In this Section some simulations are presented in order to illustrate the performance of the described optimization procedure. The receiver has an ULA with $M_r = 8$ antennas and the transmitter uses $M_t = 1$ antenna. The number of frequency points is $M_f = 128$, and the number of snapshots is $M_s = 5$. For the frequency-domain parameters, typical values often observed in channel sounding experiments are used: $\sigma_n^2 = 0.1$, $\alpha_1 = 1$, $\beta = 0.07$, and $\tau = 0.1$. The angular-domain parameters are defined as $\phi = \{60°, 120°\}$, $\kappa = \{10, 50\}$, $\epsilon = \{0.4, 0.6\}$, corresponding to two clusters in the angular domain.

One specular component is assumed to be present, and it is modeled as

$$\mathbf{s}(k) = \gamma \mathbf{c}(\varphi_R) \exp(-j2\pi k\tau), \tag{10}$$

where γ is the complex gain, $\mathbf{c}(\varphi_R)$ is the steering vector for receive azimuth angle φ_R, and τ is the normalized delay. For the simulation, the values are set as $\gamma = 3e^{j*\pi/5}$, $\varphi_R = 80°$, and $\tau = 0.1$.

The received signal is generated as

$$\mathbf{y}(k) = \mathbf{s}(k) + \mathbf{R}^{1/2}\mathbf{n}_2(k) + \mathbf{n}(k), \tag{11}$$

where $\mathbf{R}^{1/2}$ is obtained by the Cholesky decomposition of $\mathbf{R}_\mathbf{y}$, $\mathbf{n}_2(k)$ is a circular complex white Gaussian process.

The iterative procedure described in Section III is repeated 5 times, starting with the estimation of the specular component and proceeding as indicated in Figure 1. In Figure 2 we compare the power delay profile (PDP) and power angular profile (PAP) obtained using the estimation procedure described in this article with the actual PDP and PAP, respectively. The curves overlap almost perfectly. The estimator provides high-precision estimates for the time-delay distribution, angular distribution, and specular component. The PAP is compared to the output of the Bartlett beamformer, showing the gain in using the combined procedure to estimate each signal component iteratively. The beamformer is only able to estimate the angle of the specular component, but can not provide any useful information about the diffuse scattering component.

V. Conclusion

In this paper we derive an estimator that jointly estimates the parameters of the concentrated propagation paths and the distributed scattering component. We propose a two step procedure that alternates between the estimation of the parameters of the concentrated propagation paths and the parameters of the distributed scattering.

The joint angular-delay model for the distributed scattering leads to a correlation matrix with high dimensionality, which prevents direct implementation of a maximum-likelihood (ML) estimator. In this paper, computationally efficient methods for finding the ML estimates are derived. The Kronecker and Toeplitz structures of the covariance matrices are fully exploited.

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