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## **ESTIMATION OF MULTIDIMENSIONAL POLARIMETRIC CHANNEL MODEL PARAMETERS**

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# ESTIMATION OF MULTIDIMENSIONAL POLARIMETRIC CHANNEL MODEL PARAMETERS

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**Abstract:** We describe an application of the SAGE algorithm for parameter estimation, which is suitable for MIMO channel sounding using polarization sensitive antenna arrays at both link ends (mobile station (MS) and base station (BS)). To adapt the SAGE algorithm for this application a description of the polarization dependent double directional time variant complex transfer matrix is given. Based on this model the SAGE algorithm allows a joint estimation of the four complex polarization transmission path weights, the directions of arrival, directions of departure, time delays and the Doppler-frequencies of waves propagating in mobile radio environments. Finally the estimation results of first measurements are presented showing the polarization dependent wave propagation in a mobile radio channel.

## 1 Introduction

The interest in the multidimensional structure of the mobile radio channel is growing rapidly. The initial motivation was to describe the space-time structure at the base station. In the recent time a more detailed description of the mobile radio channel is required for several reasons. At one hand it is important to get a better understanding of the wave propagation mechanism in real radio environments. With this knowledge it is possible to develop MIMO-systems, which can profit from space and time diversity. Using the dimension polarization the capacity of such systems will be probably increased. For example it is easier to integrate two antennas with orthogonal polarization (patch antenna) in a dense space than two antennas with the same polarization, which can profit only from the space diversity. A second reason for our investigations is the possibility of the better de-embedding of the measuring antennas at both link ends for link-level-simulations with different types of antennas. In the first section a simplified model of the mobile radio channel taking polarization into account is introduced. The second section proposes a Maximum Likelihood Estimator (ML) based on the SAGE (Space Alternating Generalized Expectation Maximization) algorithm for the channel parameter extraction. These estimated parameters can be used for the initialization of a gradient based ML estimator, which converges faster for closely spaced path. Finally we show some results of a first measurement using the advantage of polarization sensitive antennas.

## 2 Simplified Structure of the Mobile Radio Channel

The propagation of the electromagnetic wave between two points (BS and MS) can be described through the superposition of  $K$  discrete propagation paths. Each path  $k$  is characterized by its angles of departure elevation  $\vartheta_{Tx,k}$  and azimuth  $\varphi_{Tx,k}$

$$\Omega_{Tx,k}(\vartheta_{Tx,k}, \varphi_{Tx,k}) \quad (2.1)$$

angles of arrival

$$\Omega_{\text{Rx},k}(\vartheta_{\text{Rx},k}, \phi_{\text{Rx},k}), \quad (2.2)$$

time delay

$$\tau_k, \quad (2.3)$$

Doppler frequency,

$$\alpha_k \quad (2.4).$$

and the four complex polarization transmission path weights

$$\underline{\gamma}_k = \begin{bmatrix} \gamma_{\text{HH},k} \\ \gamma_{\text{VH},k} \\ \gamma_{\text{HV},k} \\ \gamma_{\text{VV},k} \end{bmatrix} \quad (2.5)$$

A real antenna transmits or receives always a superposition of vertical and horizontal components. That's why the antenna symbol in Fig. 2.1 includes these two components for only one real antenna. Therefore the complex transfer vector  $\underline{\gamma}_k$  describes the four possible combinations between two antennas (example Fig. 2.1) or antenna arrays for one path.

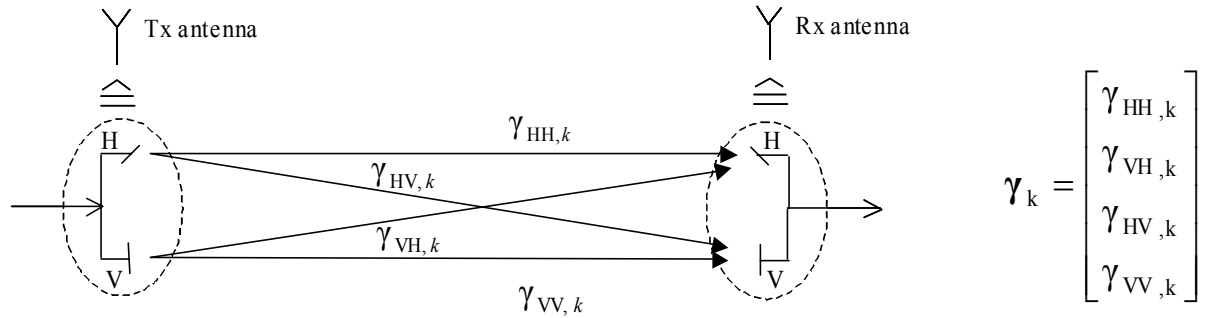


Fig. 2.1 complex transfer vector

### 3 Extended Structure of the Mobile Radio Channel

The measured complex transfer matrix  $\mathbf{H}$  of the mobile radio channel is dependent on the used transmit and receive antenna arrays and on the used measurement system. For this case a description of the polarization dependent double directional time variant complex transfer matrix follows. In order to estimate the described parameters of the mobile radio channel it is necessary to use polarization sensitive antenna arrays, which antennas are characterized by its beam patterns  $\mathbf{b}_H$  and  $\mathbf{b}_V$  for horizontal and vertical stimulation. The function  $\mathbf{B}_A(\Omega_{\text{Rx}}, \Omega_{\text{Tx}})$  combines the characteristic from the transmitter ( $M_{\text{Tx}}$  antennas) and receiver ( $M_{\text{Rx}}$  antennas) side, (whereby superscript  $[\cdot]$  denotes the dimension of a matrix or vector)

$$\begin{aligned} \mathbf{B}_A(\Omega_{\text{Tx}}, \Omega_{\text{Rx}})^{[M_{\text{Tx}} \cdot M_{\text{Tx}} \times 4]} &= [\mathbf{b}_{\text{Tx},H}(\Omega_{\text{Tx}}) \quad \mathbf{b}_{\text{Tx},V}(\Omega_{\text{Tx}})]^{[M_{\text{Tx}} \times 2]} \otimes [\mathbf{b}_{\text{Rx},H}(\Omega_{\text{Rx}}) \quad \mathbf{b}_{\text{Rx},V}(\Omega_{\text{Rx}})]^{[M_{\text{Rx}} \times 2]} \\ &= \mathbf{B}_{\text{Tx}} \otimes \mathbf{B}_{\text{Rx}} \end{aligned} \quad (3.1)$$

The measurement system with the system bandwidth of  $B = (M_f - 1) \cdot \Delta f$  itself has the frequency response  $\mathbf{G}_f$ . The complete frequency response of the channel and the measurement system is:

$$\mathbf{b}_f(\tau_k) = \mathbf{G}_f \cdot e^{-j2\pi\tau_k(f_0 + \Delta f \cdot \mathbf{m}_f)}; \quad \mathbf{m}_f = \left[ -\frac{M_f - 1}{2} \quad \dots \quad \frac{M_f - 1}{2} \right]^T \quad (3.2)$$

The short time variance (only phase) of  $\gamma_k$  can be described by

$$\mathbf{b}_t(\alpha_k) = e^{-j2\pi\alpha_k(t_0 + \Delta t \cdot \mathbf{m}_t)}; \quad \mathbf{m}_t = \left[ -\frac{M_t - 1}{2} \quad \dots \quad \frac{M_t - 1}{2} \right]^T. \quad (3.3)$$

Therefore the polarization dependent double directional time variant complex transfer matrix can be defined by

$$\mathbf{H}^{[M \times 1]} = \sum_{k=0}^K \left\{ \mathbf{b}_t(\alpha_k) \otimes \mathbf{b}_f(\tau_k) \otimes \mathbf{B}_{Tx}(\Omega_{Tx,k}) \otimes \mathbf{B}_{Rx}(\Omega_{Rx,k}) \right\} \cdot \gamma_k = \sum_{k=0}^K \mathbf{B}(\boldsymbol{\mu}_k) \cdot \gamma_k \quad (3.4)$$

$$M = M_t \cdot M_f \cdot M_{Tx} \cdot M_{Rx}; \quad \boldsymbol{\mu}_k = \left[ \alpha_k \quad \tau_k \quad \Omega_{Tx,k} \quad \Omega_{Rx,k} \right]$$

The complex transfer vector  $\gamma_k$  (2.5) is in general frequency dependent, but in the narrow band case it can be treated as frequency independent.

#### 4 MAXIMUM LIKELIHOOD ESTIMATION

In this section the Maximum Likelihood (ML) estimator SAGE (Space Alternating Generalized Expectation Maximization) [1] for the polarization dependent double directional time variant complex transfer matrix is explained. One intention to use this algorithm is the possibility to use circular as well as linear arrays during channel measurements. In [2] we have shown that there are problems with convergence speed of the SAGE algorithm for closely spaced path. A gradient based ML estimator was developed to solve this problem [2], but it is still necessary to initialize this algorithm. This section describes the SAGE algorithm in general and furthermore a way of an effective implementation. The measured data our observation (corresponding to (3.4)) is the superposition of all  $K$  path components  $\mathbf{s}_k^{[M \times 1]}$  and the noise, which is independent identical normal distributed in real and imaginary part with a standard deviation of  $\sigma_r$ .

$$\mathbf{x}^{[M \times 1]} = \mathbf{s}(\boldsymbol{\mu})^{[M \times 1]} + \sigma_r \cdot \mathbf{n}^{[M \times 1]} \quad (4.1)$$

$$\mathbf{s}(\boldsymbol{\mu}) = \sum_{k=1}^K \mathbf{s}_k(\boldsymbol{\mu}_k) = \sum_{k=1}^K \mathbf{B}_k(\boldsymbol{\mu}_k) \cdot \gamma_k \quad (4.2)$$

The objective of the ML algorithms is to find maximum of the log likelihood function, but in our case we can even say we have to find the global maximum of the non linear least squares problem

$$\hat{\boldsymbol{\mu}}_{opt} = \underset{\boldsymbol{\mu}}{\operatorname{argmax}} \left\{ \varepsilon^2(\mathbf{x}, \boldsymbol{\mu}) \right\} \quad (4.3)$$

with

$$\varepsilon^2(\mathbf{x}, \boldsymbol{\mu}) = -(\mathbf{x} - \hat{\mathbf{s}}(\boldsymbol{\mu}))^H \cdot (\mathbf{x} - \hat{\mathbf{s}}(\boldsymbol{\mu})) \quad (4.4)$$

Since the full search for  $\hat{\boldsymbol{\mu}}_{opt}$  is of exponential complexity the SAGE algorithm reduces the  $6K$  multidimensional search to  $K$  independent line search problems. It updates the  $L=6$  parameters  $[\alpha_k \ \tau_k \ \theta_{Tx,k} \ \varphi_{Tx,k} \ \theta_{Rx,k} \ \varphi_{Rx,k}] \triangleq [\mu_{k,1} \ \dots \ \mu_{k,6}]$  successively (Fig. 4.1). The main part of the SAGE algorithm is the search of the maximum in a correlation function  $\mathbf{c}(\mathbf{r}, \mu_{k,l})$  dependent on only one parameter  $\mu_{k,l}$  (all other parameters are taken as constant). Whereby  $\mathbf{r}$  denotes the remaining part of the observation  $\mathbf{x}$  after removing all other estimated paths.

$$\mathbf{r} = \mathbf{x} - \sum_{p=1, p \neq k}^K \hat{\mathbf{s}}_p = \mathbf{x} - \sum_{p=1, p \neq k}^K \mathbf{B}_p(\hat{\boldsymbol{\mu}}_p) \cdot \hat{\boldsymbol{\gamma}}_p \quad (4.5)$$

The correlation function for each parameter results from the cost function

$$\begin{aligned} \Lambda^2(\mathbf{r}, \mu_{k,l}) &= -(\mathbf{r} - \mathbf{s}(\mu_{k,l}))^H \cdot (\mathbf{r} - \mathbf{s}(\mu_{k,l})) \\ &= -\mathbf{r}^H \cdot \mathbf{r} + \mathbf{s}(\mu_{k,l})^H \cdot \mathbf{r} + \mathbf{r}^H \cdot \mathbf{s}(\mu_{k,l}) - \mathbf{s}(\mu_{k,l})^H \cdot \mathbf{s}(\mu_{k,l}) \end{aligned} \quad (4.6)$$

depending on  $\mu_{k,l}$ . Eliminating  $\mathbf{r}^H \cdot \mathbf{r}$ , which is independent from the parameter  $\mu_{k,l}$  and substituting  $\mathbf{s}(\mu_{k,l})$  by:

$$\mathbf{s}(\mu_{k,l}) = \mathbf{B}_k(\mu_{k,l}) \cdot \boldsymbol{\gamma}_k \quad (4.7)$$

with (whereby  $\mathbf{B}^+$  denotes the pseudo inverse matrix of  $\mathbf{B}$ )

$$\boldsymbol{\gamma}_k = \mathbf{B}_k(\mu_{k,l})^+ \cdot \mathbf{r} \quad (4.8)$$

and

$$\mathbf{B}_k(\mu_{k,l})^+ = \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l}) \quad (4.9)$$

The correlation function of  $\mu_{k,l}$  is

$$\begin{aligned} \mathbf{c}(\mathbf{r}, \mu_{k,l}) &= \left( \mathbf{B}_k(\mu_{k,l}) \cdot \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \mathbf{r} \right)^H \cdot \mathbf{r} \\ &\quad + \mathbf{r}^H \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \mathbf{r} \\ &\quad - \left[ \left( \mathbf{B}_k(\mu_{k,l}) \cdot \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \mathbf{r} \right)^H \right. \\ &\quad \left. \cdot \left( \mathbf{B}_k(\mu_{k,l}) \cdot \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \mathbf{r} \right) \right] \end{aligned} \quad (4.10)$$

For an effective implementation some transformations are necessary, it is straight forward to see that (4.10) yields

$$\mathbf{c}(\mathbf{r}, \mu_{k,l}) = \mathbf{r}^H \cdot \mathbf{B}_k(\mu_{k,l}) \cdot \left( \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{B}_k(\mu_{k,l}) \right)^{-1} \cdot \mathbf{B}_k(\mu_{k,l})^H \cdot \mathbf{r} \quad (4.11)$$

Using the QR decomposition of

$$\mathbf{B}_k(\mu_{k,l}) = \mathbf{Q}_k(\mu_{k,l}) \cdot \mathbf{R}_k(\mu_{k,l}) \quad (4.12)$$

the correlation function can be expressed as

$$\mathbf{c}(\mathbf{r}, \mu_{k,l}) = \mathbf{r}^H \cdot \mathbf{Q}_k(\mu_{k,l}) \cdot \mathbf{Q}_k(\mu_{k,l})^H \cdot \mathbf{r} \quad (4.13).$$

The calculation of  $\mathbf{Q}_k(\mu_{k,l})$  is expensive but it is possible to resolve the problem into 4 independent QR decompositions

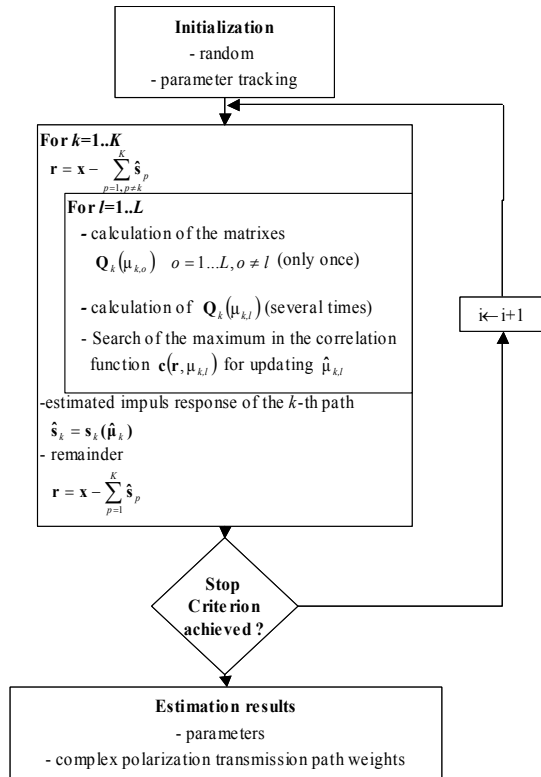
$$\mathbf{Q}_k(\mu_{k,l}) = \mathbf{q}_l(\alpha_k) \otimes \mathbf{q}_f(\tau_k) \otimes \mathbf{Q}_{\text{Tx}}(\Omega_{\text{Tx},k}) \otimes \mathbf{Q}_{\text{Rx}}(\Omega_{\text{Rx},k}) \quad (4.14)$$

In order to calculate the correlation function for  $\mu_{k,l}$  the QR decomposition for the three from  $\mu_{k,l}$  independent Q matrixes has to be done only once. The QR decomposition for the matrix Q which depends on  $\mu_{k,l}$  has to be calculated several times which actually depends on the number of required samples in the correlation function. With the simplified structure of (4.10) the estimated parameter  $\mu_{k,l}$  is:

$$\hat{\mu}_{k,l} = \underset{\mu_{k,l}}{\operatorname{argmax}} \{ \mathbf{c}(\mathbf{r}, \mu_{k,l}) \} = \underset{\mu_{k,l}}{\operatorname{argmax}} \{ \mathbf{r}^H \cdot \mathbf{Q}_k(\mu_{k,l}) \cdot \mathbf{Q}_k(\mu_{k,l})^H \cdot \mathbf{r} \} \quad (4.15)$$

The four complex polarization transmission path weights result from the least squares solution of

$$\begin{bmatrix} \hat{\gamma}_1 \\ \vdots \\ \hat{\gamma}_K \end{bmatrix} \approx [\mathbf{B}(\hat{\mu}_1) \quad \cdots \quad \mathbf{B}(\hat{\mu}_K)]^+ \cdot \mathbf{x} \quad (4.16)$$



Before the first iteration of the algorithm all parameters  $\hat{\mu}$  have to be initialized. There are several possibilities. The simplest method is to start with one path, whereby its parameters  $\hat{\mu}$  set to zero or a random value. For measurement drives it is also possible to initialize with the estimation results of a previous snapshot (parameter tracking). The estimation results of the SAGE algorithm can be used for a gradient based ML estimator because of the slow convergence [2] of the SAGE algorithm for closely spaced path. In this case a stop criterion is just a certain number of iterations.

Fig. 4.1 flow graph of the SAGE algorithm

## 5 Measurement

In this section first result of a MIMO-measurement with polarization sensitive arrays are shown. A vector channel sounder described in [3] was used. The receive array was a uniform circular patch array with 24 polarization sensitive patch antennas (Fig. 5.1 (b)) and on the transmit side a dual polarized omni directional antenna (Fig. 5.1 (a)) was used.

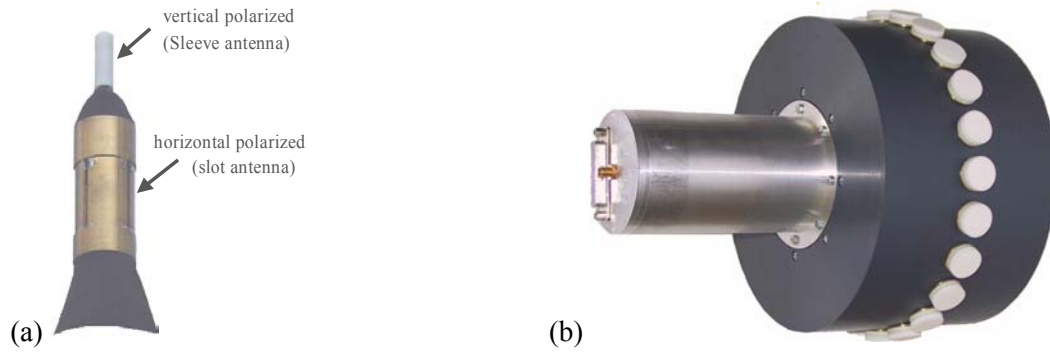


Fig. 5.1 (a) transmit array, (b) receive array (PUCPA)

In order to demonstrate the correctness of the algorithm and the used model of the mobile radio channel some simple measurements were done in the court yard of our department. The parameters azimuth of arrival and the time delay were estimated by the described algorithm. The location of the receive array is labeled with Rx (Fig. 5.2), the transmit antenna was moved from position Tx1 across Tx7 to Tx8. In Fig. 5.2 (a) and (b) the estimation result of 20 snapshots close to Tx7 for two different estimation setups are plotted. In the first case both ports horizontal and vertical polarization were used for the joint estimation of the mentioned parameters plus the complex polarization transmission path weights, but in the second case just the vertical port of the transmit array was used for the estimation. In both setups it is possible to identify the paths, which are reflections on the walls. The reflections caused by the sculpture can not be found in case of using only one polarization on the transmit side, thereby it was not important whether the horizontal or the vertical port was stimulated. Not until the joint estimation of all parameters and path weights this path could be identified.

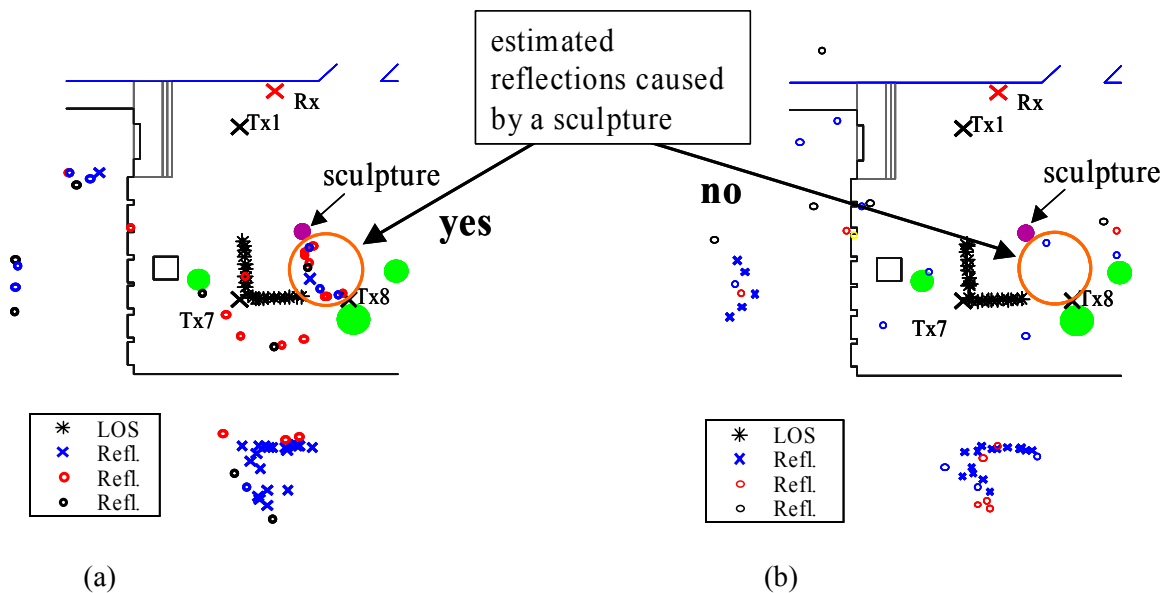


Fig. 5.2 (a) joint estimation of all pol. trans. path weights, (b) only vertical stimulation on Tx side

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## 6 Conclusion

The correctness of the described application of the SAGE algorithm is demonstrated on estimation results of a measurement. The results show that there is an advantage of using polarization sensitive antenna arrays in order to describe the mobile radio more detailed as up to now. But there is still a lot of work to do for a better understanding of the wave propagation in different scenarios of a real mobile radio environments.

## 7 References

- [1] Klaus I.Pedersen, Bernard H.Fleury and Preben.Mogensen, High Resolution of Electromagnetic Waves in Time- Varying Radio Channels, Nachdiplomarbeit, IEEE 1997, Center of PersonKommunikation(CPK), Aalborg University, Denmark
- [2] Andreas Richter, Markus Landmann, and Reiner S. Thomä, A Gradient Based Method for Maximum Likelihood Channel Parameter Estimation from Multidimensional Channel Sounding Measurements, XXVIIth URSI-GA, August 18-24, Maastricht, Ilmenau University of Technology, Department of Electrical Engineering and Information Technology
- [3] R.S. Thomä, D.Hampicke, A.Richter, G. Sommerkorn, A.Schneider, U.Trautwein and W. Wirnitzer, Identification of time-variant directional mobile radio channels, IEEE Trans.Instrum. and Meas., Vol.49, Apr 2000, pp.357-364, Ilmenau University of Technology, Department of Electrical Engineering and Information Technology