

DoA Resolution Limits in MIMO Channel Sounding

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Abstract: Several high-resolution parameter estimation algorithms have been applied to directional channel parameter estimation in channel sounding. However, variance and reliability of the estimation results are not often clearly defined in practical environments. Especially, imperfections, mutual coupling, and residual calibration errors of real antenna arrays impose limits to the DoA parameter estimation performance. One critical example is the resolution of coherent wave fronts which is often the case in real-time channel sounding. We derive the Cramer-Rao-Lower-Bound (CRLB) of an unbiased direction-of-arrival (DoA) estimator. The CRLB indicates the minimum achievable parameter variance which can not be outperformed by any parameter estimator. It is shown that the required derivatives of the observations with respect to the DoA and path weight parameters can be deduced from a properly defined calibration data model.

1 Introduction

For optimum design and performance evaluation of transmission systems using antenna arrays and sophisticated space-time algorithms we need realistic directional channel models. MIMO channel sounding observations [1] [2] are characterized by DoA and DoD, the time delay and the Doppler shift of any relevant propagation path. Furthermore, any path is described by a 2×2 polarimetric complex path weight matrix. This parametric path model and the consideration of diffuse components [3] roughly depicts the underlying data model which is used for high-resolution parameter estimation from measurements. Several high-resolution channel parameter estimation algorithms have been proposed for this application, mainly based on subspace methods [4] or maximum likelihood algorithms [5] [6] [7] [8]. However, resolution and reliability of these estimation procedures are not often clearly defined. It has been observed, that a high-resolution channel parameter estimator may yield results that approximates the observed data very well. Although, closer inspection of the data often shows that there is no physical relevance. One typical example is that a small physical path may be approximated by two paths with nearly equal magnitude and opposite signs, leading to physically meaningless weights (line splitting). Apparently, this is often a result of model order over-estimation. It was also observed that the variance of this estimates is typically very high. The most severe problems arise if coherent wave-fronts are involved which is in general the case for real-time channel sounding where we aim to estimate one set of multidimensional parameters from only one single snap-shot (see also [4] [10]). Since resolution and reliability of high-resolution parameter estimation has always something to do with calibration or precise knowledge of the measurement device, it seems quite clear that the quality of the antenna arrays may be considered as the weakest point in the performance of high-resolution channel sounding. The performance of a real antenna array is always susceptible to various imperfections, mutual element coupling and residual calibration errors. Moreover, the observed spatial aperture, which has strong influence on resolution, is severely limited by the available array size and data model constraints.

In this paper we will derive the fundamental limitations on the achievable DoA and path weight variance in terms of the Cramer-Rao-Lower-Bound (CRLB) [11] [12] of an unbiased DoA parameter estimator. The CRLB indicates the minimum achievable parameter variance which can not be outperformed by any parameter estimator. The advantage of our method is that it completely relies on measured antenna characteristics. Therefore we also introduce a calibration data model that has the advantage to deliver the required derivatives of the observations with respect to the DoA and path weight parameters which we need for the Fisher information matrix calculation.

2 Cramer-Rao-Lower-Bound (CRLB)

The CRLB establishes a lower bound on the variance of any unbiased channel parameter estimator [11]. The CRLB on the variances and covariances of the channel parameters such as angle of arrival (resp. departure) $\Omega(\vartheta, \varphi)$ and of the complex path weights γ are defined as the elements of the inverse Fisher-Information-Matrix (FIM). The FIM is given by the covariance matrix of the L first order derivatives of the observed data vector with respect to the parameters θ of the K paths in the given scenario: $\theta^{(k)} = [\vartheta_1 \dots \vartheta_K] [\varphi_1 \dots \varphi_K] [\text{Re}\{\gamma_1\} \dots \text{Re}\{\gamma_K\}] [\text{Im}\{\gamma_1\} \dots \text{Im}\{\gamma_K\}]$. The angles elevation ϑ and azimuth φ are defined in the spherical coordinate system. The observed data of the channel can be expressed by

$$\mathbf{x}^{(M)}(\theta, \mathbf{n}) = \mathbf{s}^{(M)}(\theta) + \mathbf{n}^{(M)}$$

whereby \mathbf{n} denotes the noise vector and

$$\mathbf{s}(\theta) = \sum_{k=1}^K \gamma_k \cdot \mathbf{b}(\Omega_k)$$

results from the superposition of all K paths weighted by the according complex polarimetric array response $\mathbf{b}(\Omega_k)$. For a known θ the probability density function of the observation \mathbf{x} is:

$$pdf(\mathbf{x} | (\theta, \mathbf{R}_m)) = \frac{1}{\pi^M \det(\mathbf{R}_m)} e^{-(\mathbf{x}-\mathbf{s}(\theta))^H \mathbf{R}_m^{-1} (\mathbf{x}-\mathbf{s}(\theta))}$$

To calculate the CRLB of a real antenna array with respect to the parameters azimuth φ , and elevation ϑ we need the complex polarimetric array response $\mathbf{b}(\Omega_k)$ and its derivatives. To this end we define an antenna data model which maps the measured M_{R_x, T_x} beam patterns $\mathbf{b}(\varphi, \vartheta)$ to what we call the effective aperture distribution function (EADF) $\mathbf{g}(f_1, f_2)$. Actually, due to the periodic nature of the beam patterns in azimuth and elevation, the EADF can be considered as a two-dimensional Fourier series expansion of the beam pattern. An advantage of this approach is that the EADF typically is concentrated to a very small support area. Another advantage is that the derivatives can be calculated algebraically. The model describes the array manifold for a certain frequency f or for some narrow bandwidth around a center frequency f_0 . The complex polarimetric beam patterns $\mathbf{b}(\Omega_k)$ [7] [8] are measured in a well defined propagation environment which should be an anechoic chamber, whereby the pivot point of the antenna array is located in the origin of the spherical coordinate system. The measured beam patterns are discrete in azimuth $\varphi = (-\pi \dots \Delta\varphi \dots \pi - \Delta\varphi)$ and elevation $\vartheta = (0 \dots \Delta\vartheta \dots \pi)$. The beam pattern $\mathbf{b}(\varphi, \vartheta, m)$ of the antenna m is stored in the matrix $\mathbf{B}^{(N_x, N_y)}$ (the superscript $[\cdot]$ denotes the dimension of the matrix). If the azimuth response is measured in $[0, 2\pi)$, elevation characterization in $[0, \pi]$ is enough to completely describe the spherical beam pattern. However, for two-dimensional signal processing this is not convenient and we have to construct a fully 2D-periodic data structure by some periodic extension. The 2D-periodic beam pattern $\mathbf{B}_p^{(N_x, N_y)}$ is defined as

$$\mathbf{B}_p^{(N_x, N_y)} = \begin{bmatrix} \mathbf{B} \\ \mathbf{B}_r \end{bmatrix}; N_x = \left(\frac{\pi}{\Delta\vartheta} + 1\right); N_y = \frac{2\pi}{\Delta\varphi}; N_x = 2(N_y - 1)$$

Half of the data of $\mathbf{B}_p^{(N_x, N_y)}$ is redundant, since the matrix $\mathbf{B}_r^{(N_x, N_y)}$ is $-\mathbf{B}$ shifted by 180° in azimuth and flipped in elevation direction. This is caused by the redundancy of spherical coordinate system when elevation crosses the poles (0° and 180° elevation). In essence, it is a shift of 180° in azimuth and a rotation of the polarization vector by 180 degrees. By discrete Fourier transform using the elevation transformation matrix

$$\mathbf{F}_1 = e^{-j2\pi\vartheta'f_1} = e^{-j2\pi\vartheta\vartheta'f_1/\Delta\vartheta} = e^{-j2\pi\vartheta\vartheta'f_1/N_1}; \Delta\vartheta' = \frac{2\pi}{N_1}; \Delta f_1 = \frac{1}{2\pi}; \vartheta' = \vartheta - \pi$$

with $\mathbf{n}_1 = \mu_1^T = \left[\frac{2\pi}{N_1} \dots \frac{2\pi}{N_1} - 1 \right]$ and the azimuth transformation matrix

$$\mathbf{F}_2 = e^{-j2\pi\varphi'f_2} = e^{-j2\pi\varphi\varphi'f_2/\Delta\varphi} = e^{-j2\pi\varphi\varphi'f_2/N_2}; \Delta\varphi = \frac{2\pi}{N_2}; \Delta f_2 = \frac{1}{2\pi}$$

with $\mathbf{n}_1^T = \boldsymbol{\mu}_1 = [-\frac{N_1}{2} \dots \frac{N_1}{2} - 1]$ we get the EADF of the m -th antenna

$$\mathbf{G}_m = \frac{1}{\sqrt{N}} \mathbf{F}_1 \cdot \mathbf{B}_p \cdot \mathbf{F}_2; \quad N = N_1 \cdot N_2.$$

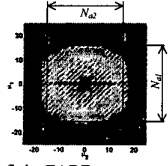


Fig. 1 finite EADF support

In case of over-sampling of the beam pattern in the angular domain this transformation maybe used to achieve a data compression. The required size $N_s = N_{s1} \cdot N_{s2}$ of the effective aperture is determined by the beam width of the antenna and the array size (Fig. 1). For the marginal case of the isotropic radiator the effective aperture is just a Dirac impulse. Only the data of the finite support area are kept for further processing.

The complex exponentials for the EADF transformed to the beam domain are defined for elevation by $\mathbf{d}_{s1}(\vartheta) = e^{j\boldsymbol{\mu}_1^T \boldsymbol{\vartheta}}$ and azimuth by $\mathbf{d}_{s2}(\varphi) = e^{j\boldsymbol{\mu}_2^T \boldsymbol{\varphi}$ with $\boldsymbol{\mu}_1^T = [-\frac{N_1}{2} \dots \frac{N_1}{2}]$ and $\boldsymbol{\mu}_2^T = [-\frac{N_2}{2} \dots \frac{N_2}{2}]$. The beam pattern and its derivatives for an arbitrary azimuth/elevation angle pair $\Omega(\vartheta, \varphi)$ are given by the following equations:

$$\mathbf{b}(\vartheta, \varphi, m) = \frac{1}{\sqrt{N_s}} \mathbf{d}_{s1} \cdot \mathbf{G}_m \cdot \mathbf{d}_{s2}$$

$$\frac{\partial \mathbf{b}(\vartheta, \varphi, m)}{\partial \vartheta} = \frac{1}{\sqrt{N_s}} j \cdot \mathbf{d}_{s1} \cdot \text{diag}\{\boldsymbol{\mu}_1^T\} \cdot \mathbf{G}_m \cdot \mathbf{d}_{s2}$$

$$\frac{\partial \mathbf{b}(\vartheta, \varphi, m)}{\partial \varphi} = \frac{1}{\sqrt{N_s}} j \cdot \mathbf{d}_{s1} \cdot \mathbf{G}_m \cdot \text{diag}\{\boldsymbol{\mu}_2^T\} \cdot \mathbf{d}_{s2}$$

We get the equations for an antenna array with M antennas by $\Gamma^{[N_s \times N_s]} = [\text{vec}\{\mathbf{G}_1\} \dots \text{vec}\{\mathbf{G}_M\}]^T$ and $\mathbf{d}_s^{[N_s \times N_s]}(\vartheta, \varphi) = \text{vec}\{\mathbf{d}_{s1}^T(\vartheta) \cdot \mathbf{d}_{s2}^T(\varphi)\}$:

$$\mathbf{b}(\vartheta, \varphi)^{[N_s \times 1]} = \frac{1}{\sqrt{N_s}} \cdot \Gamma \cdot \mathbf{d}_s.$$

The derivatives of the beam patterns $\mathbf{b}(\vartheta, \varphi)$ with respect to the azimuth and elevation angle are calculated equivalently using the derivatives of the complex exponentials $\frac{\partial \mathbf{d}_{s1}(\vartheta, \varphi)}{\partial \vartheta}$ and $\frac{\partial \mathbf{d}_{s2}(\vartheta, \varphi)}{\partial \varphi}$ instead of \mathbf{d}_s . The derivatives with respect to the real and imaginary part of the parameters of the k -th path weight are

$$\frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Re}\{\gamma_k\}} = \mathbf{b}(\Omega_k) \text{ and } \frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Im}\{\gamma_k\}} = j \cdot \mathbf{b}(\Omega_k).$$

The matrix of the first order derivatives is given by

$$\mathbf{A} = \begin{bmatrix} \frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Re}\{\gamma_1\}} & \dots & \frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Re}\{\gamma_L\}} \\ \frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Im}\{\gamma_1\}} & \dots & \frac{\partial \mathbf{b}(\boldsymbol{\theta})}{\partial \text{Im}\{\gamma_L\}} \end{bmatrix}.$$

The real part of the Gramian of \mathbf{A} is proportional to the Fisher-Information-Matrix (FIM):

$$\mathbf{J} = 2 \cdot \text{Re}\{\mathbf{A}^H \cdot \mathbf{R}_{\text{noise}}^{-1} \cdot \mathbf{A}\}$$

whereby the diagonal elements of the inverse FIM represent the variances of the L parameters

$$\text{CRLB}(\boldsymbol{\theta}) = \mathbf{J}^{-1}$$

and the off diagonal elements denote the co-variances. In case of uncorrelated parameters these off diagonal elements are zero. With a single path test scenario it is possible to calculate the maximum possible resolution for a given SNR. Normally the off diagonal elements are small in this case. Because of the limited spatial aperture and imperfections of the antenna array in more complicated scenarios with two or more coherent paths the off diagonal elements will not disappear. This indicates the coupling between the parameters of the different paths and will increase the overall variances of the path parameters.

3 Conclusions

The described performance evaluation method based on the CRLB allows to evaluate arbitrary antenna arrays in single and multiple path scenarios. The performance measure proposed can be used to compare two antenna arrays on a fair basis, since it not depend on a parameters estimator. The method only requires a good calibration measurement of the array in a anechoic chamber.

This approach can also be used to calculate the variance of parameter estimates throughout high-resolution channel parameter estimation. This is of specific interest in coherent multi-path

scenarios. Here the paths are often close in terms of the maximum possible resolution since the resulting variance may be rather big depending on the phase difference between the paths. Unreliable results are indicated by big variances, which can be determined by the proposed method.

4 References

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